UDC 621.311.2:536.755

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OPTIMAL FREQUENCY CONTROL OF THE INDUCTION ELECTRIC DRIVE BASED ON THE THERMODYNAMICS OF IRREVERSIBLE PROCESSES

Abstract *The modeling method of steady state processes based on energy is applied for analysis and optimization of frequency controlled induction electric drive. This made it possible to easily determine the conjunction degree between input and output of electromagnetic part of induction motor, take into account losses in steel and copper at different frequency and voltage and simply define the optimal from energy point of view motor slip on the criterion of maximum thermodynamic efficiency.*

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ОПТИМАЛЬНЕ КЕРУВАННЯ ЧАСТОТА ІНДУКЦІЙНОГО ЕЛЕКТРОПРИВОДУ НА ОСНОВІ ТЕРМОДИНАМІКИ НЕОБОРОТНИХ ПРОЦЕСІВ

Анотація. *Застосовано метод моделювання усталених процесів на енергетичній основі для аналізу та оптимізації частотно регульованого асинхронного електропривода. Це дало змогу легко визначати ступінь спряженості між входом та виходом електромагнітної частини асинхронного двигуна, врахувати втрати в сталі і міді за різних значень частоти і напруги живлення і просто визначати за критерієм максимальної термодинамічної ефективності оптимальне з енергетичної точки зору ковзання.*

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ОПТИМАЛЬНОЕ УПРАВЛЕНИЕ ЧАСТОТА ИНДУКЦИОННОГО ЭЛЕКТРОПРИВОДА НА ОСНОВЕ ТЕРМОДИНАМИКИ НЕОБРАТИМЫХ ПРОЦЕССОВ

Аннотация *Примененен метод моделирования установившихся процессов на энергетической основе для анализа и оптимизации частотно регулируемого асинхронного электропривода. Это позволило легко определять степень сопряженности между входом и выходом электромагнитной части асинхронного двигателя, учесть потери в стали и меди при разных значениях частоты и напряжения питания и просто определять по критерию максимальной термодинамической эффективности оптимальное с энергетической точки зрения скольжение.*

Introduction. Energy is the driving force of any process, and patterns of its conversion determine the effectiveness of processes [3]. Consequently, there is growing popularity of energy approaches to mathematical description, research, computer modeling of static and dynamic processes in systems of different types: bond-graph modeling, behavioral modeling, passivity-based control, energy-shaping approach etc. [8-10]. One of these areas, based on the fundamentals of thermodynamics of irreversible processes (TDIP), is developed in application to electromechanical systems [5,7]

Analysis of recent findings and publications. The stationary operating modes of the DC motors of different types of excitation under feeding from the voltage and current sources were described on the TDIP basis [4,6]. The introduction of the DS motor as a basic power converter (PC) made it possible to obtain a number of universal relationships that characterize the optimal modes of its operation.

The induction machine (IM) is a fundamentally different electromechanical object, for which energy efficiency is very important because of its wide use. To base the working point of the IM in various operating conditions, quite complicated optimization algorithms are used [2]. An alternative to this could be energy optimization based on the TDIP.

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The purpose of the paper is to provide the mathematical description and theoretical analysis of the frequency controlled IM by the TDIP and to develop methods of energy optimization for the operating modes of the induction electric drive.

The main material. Our way of mathematical modeling of the steady state processes is a decomposition of the individual subsystems that can be presented as basic quadripoles with the parameters of power-flow (respectively X and J) at the input (i) and output (o):

$$\begin{cases} J_{\mathrm{i}} = L_{\mathrm{ii}} X_{\mathrm{i}} + L_{\mathrm{io}} X_{\mathrm{o}} \\ J_{\mathrm{o}} = L_{\mathrm{oi}} X_{\mathrm{i}} + L_{\mathrm{oo}} X_{\mathrm{o}} \end{cases}, \qquad (1)$$

where the kinetic coefficients (conductivity) are expressions of

$$L_{\mathrm{jk}} = \left(\frac{\partial J_{\mathrm{j}}}{\partial X_{\mathrm{k}}} \right)_{X_{\mathrm{j}} = \mathrm{const}}, \qquad (2)$$

and according to the Onsager's reciprocity principle .

The forces and their corresponding flows are selected so that their product would correspond to the power. Quadripoles are regarded as power converters (PC) without internal energy sources because the mutual kinetic coefficients are negative.

For a linear system, the value of kinetic coefficients can be found though non-working and short circuit experiments such as in passive quadripoles in theoretical electrical engineering. However, in contrast to passive quadripoles, PC are described by the unified minimum number of parameters in relative units. In our approach, borrowed from bioenergetics [1], the number of parameters is reduced to two: the primary one – conjunction factor *q* between input and output processes (*q* = 0 ... -1) and the auxiliary one – factor Z, which is used to bring the flows and forces to dimensionless units:

$$q = \frac{L_{\mathrm{io}}}{\sqrt{L_{\mathrm{ii}} L_{\mathrm{oo}}}}; \quad Z = \sqrt{\frac{L_{\mathrm{oo}}}{L_{\mathrm{ii}}}}. \tag{3}$$

To analyze the energy performance of the PC in a particular point, in contrast to passive quadripoles, only one parameter – the ratio of forces – is needed:

$$\chi = X_{\mathrm{o}} / X_{\mathrm{i}}. \tag{4}$$

In [1], universal characteristics of the PC are given, as well as a number of criteria that characterize the effectiveness of their work. Of course, efficiency is the most useful one for the evaluation of technical systems, and it is expressed through the given parameters as follows:

$$\eta = -\frac{J_{\mathrm{o}} X_{\mathrm{o}}}{J_{\mathrm{i}} X_{\mathrm{i}}} = -(Z\chi)\frac{q + (Z\chi)}{1 + q \cdot (Z\chi)}. \tag{5}$$

As seen from (5) and the graphical representation (Fig. 1), performance of the PC is determined practically only by two parameters: the topology parameter *q* and the normalized PC ratio of forces – the parameter that characterizes the working point. The dependencies (Fig. 1) show that for each *q,* there is the optimal value of an operating point at which maximum efficiency is achieved:

$$(Z\chi)_{\mathrm{opt-\eta}} = -\frac{q}{1 + \sqrt{1 - q^2}}, \tag{6}$$

and the maximum value of efficiency is equal

$$\eta_{\max} = (Z\chi)^2_{\mathrm{opt-\eta}}. \tag{7}$$

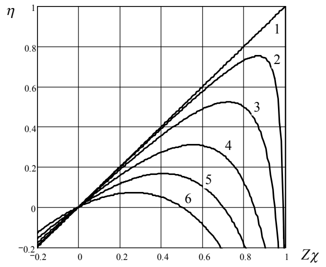

Fig.1. Dependencies of thermodynamic efficiency on the normalized ratio of forces at different levels of the conjunction factor *q*:
1– *q* = -1.0, 2– *q* = -0.99, 3– *q* = -0.95,
4– *q* = -0.85, 5– *q* = -0.7, 6– *q* = -0.5

Thus, the proposed power modeling method makes it possible to use only two parameters to assess the energy performance of operational systems or to choose the optimum parameters of the prevailing criteria.

For the transition from relative to absolute values, in addition to the two imposed PC parameters *q* and $(Z\chi)$, a third one is needed, which may be one of the kinetic coefficients L_{jk} of the system (1). Having three parameters already, such important parameters of the PC as input and output resistances (conductances) can be found [5]:

- input conductance $Y_{\mathrm{i}} = \dfrac{J_{\mathrm{i}}}{X_{\mathrm{i}}} = [1 + q \cdot (Z\chi)] L_{\mathrm{ii}}$, (8)
- output conductance $Y_{\mathrm{o}} = \dfrac{J_{\mathrm{o}}}{X_{\mathrm{o}}} = \left[1 + \dfrac{q}{(Z\chi)}\right] L_{\mathrm{oo}}$;
- pass conductances $Y_{\mathrm{oi}} = \dfrac{J_{\mathrm{o}}}{X_{\mathrm{i}}} = \left[1 + \dfrac{(Z\chi)}{q}\right] L_{\mathrm{oi}}$;

$$Y_{\mathrm{io}} = \frac{J_{\mathrm{i}}}{X_{\mathrm{o}}} = \left[1 + \frac{1}{q \cdot (Z\chi)}\right] L_{\mathrm{io}}.$$

For the equivalent T-shaped equivalent circuit of the IM, the parameters of the PC equal respectively [5]

$$q = -\frac{Z_3}{\sqrt{(Z_1 + Z_3)(Z_2 + Z_3)}}; \quad Z = \sqrt{\frac{Z_1 + Z_3}{Z_2 + Z_3}}. \tag{9}$$

The IM, according to the principles of its work, belongs to the PC with an incomplete conjunction between input and output. Modern frequency controlled induction electric drives are able to provide energy-saving modes of operation for changes in a wide range of speed and load torque on the shaft. However, determining the optimal parameters for these regimes, even in the scalar method of frequency control, is a difficult task [2]. Academician M.P. Kostenko proposed the so-called economic frequency control with a simple algorithm: for all modes the absolute slip $\beta = s f^*$ (in relative units) (*s* is the slip and $f^* = f / f_{\mathrm{n}}$ is the relative frequency of voltage) is equal to the nominal value $\beta_{\mathrm{n}} = \Delta\omega_{\mathrm{n}} / \omega_{0\mathrm{n}}$, where $\omega_{0\mathrm{n}}$ and $\Delta\omega_{\mathrm{n}}$ are respectively the non-working angular velocity and difference of angular velocity for the nominal parameters of the IM. Under this condition, the stator voltage at changing speed and load torque is regulated in a way that minimizes electromagnetic losses in the motor (total losses of energy in copper and steel). Precise power investigation of the lows of frequency controlled IM, made in [2], showed that the optimum for minimum basic electromagnetic losses is characterized by the invariance of optimal absolute slip β_{opt} from electromagnetic torque for each speed. Thus, unlike the M.P. Kostenko's economic frequency control, especially for medium and high power machines, β_{opt} is fundamentally different from β_{n} and decreases with decreasing speed. Therefore, it is interesting to study the patterns of frequency controlled IM according to the criterion of maximum energy efficiency (6)-(7).

For this study, we will use the traditional T-shaped equivalent circuit for one phase of the IM (Fig. 2), where R_1 and $L_{\sigma 1}$ are the resistance and the leakage inductance of stator windings, R_2' and $L_{\sigma 2}'$ are the same parameters of the rotor windings, led to the stator, L_{m} is the inductance of the magnetization circuit and R_{m} is the resistance that models the iron loss. Forces here are the stator voltage U_1, and led to stator winding the rotor voltage U_2' and flows – the currents I_1 and I_2' relevant.

Parameters of the T-shaped equivalent circuit of the IM as PC are determined by the expressions

$$Z_1 = R_1 + jX_{\sigma 1}; \quad Z_2 = R_2' + jX_{\sigma 2}',$$

$$Z_3 = R_{\mathrm{m}}\left[1+\left(\frac{R_{\mathrm{m}}}{X_{\mathrm{m}}}\right)^2\right]^{-1} + jX_{\mathrm{m}}\left[1+\left(\frac{X_{\mathrm{m}}}{R_{\mathrm{m}}}\right)^2\right]^{-1},$$

$$X_{\sigma 1} = 2\pi f L_{\sigma 1}; \quad X_{\sigma 2}' = 2\pi f L_{\sigma 2}'; \quad X_{\mathrm{m}} = 2\pi f L_{\mathrm{m}}. \quad (10)$$

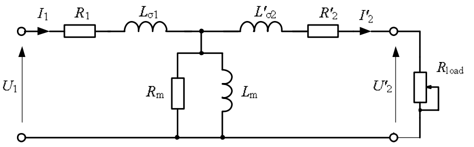

Fig.2. T- shaped equivalent circuit of the IM

Obtained through (9), the main parameters of the PC *q* and *Z* are described by complex numbers, but with a very small imaginary part. The system of equations (1) also adequately describes the PC, if all options are complex. Analysis showed that efficiency of the PC (5) would be true if it is adopted as a module of the complex number of the calculations result. The working point of maximum efficiency of the IM $(Z\chi)_{\mathrm{opt}-\eta}$ will not match the expression (6), but must be found in the studies on the extremum of function in complex variables (5). However, our studies have shown that dependence (7) is true also for this case.

For effective values of voltages and currents of the IM, the efficiency received as a module of values (5) corresponds to the correlation of full powers in output and input. For the IM, resistance $R_{\mathrm{load}} = R_2'(1-s)/s$ functions as the load of the PC that simulates the mechanical output power. So, under purely active power output, the resulting value of efficiency will be equal $\eta = \eta_{\mathrm{P}} \cos\varphi$, where η_{P} is the active power efficiency and $\cos\varphi$ is cosine of the angle shift between voltage and current at the input of the PC. The herein formulated criterion of maximum energy efficiency is a compromise between the maximums η_{P} and $\cos\varphi$.

Substituting in the second equation of the system (1) $U_2' = I_2 R_2'(1-s)/s$, we obtain

$$\chi = \frac{U_2'}{U_1} = -\frac{L_{\mathrm{io}}}{L_{\mathrm{load}} + L_{\mathrm{oo}}}, \quad (11)$$

where $L_{\mathrm{load}} = \frac{s}{1-s}\frac{1}{R_2'}$ is the conductance of the load.

From equation (11) after the transformation, the slip of the IM may be expressed as a function of the PC parameters and its working point $(Z\chi)$:

$$s = 1 - \left[1 - R_2' L_{\mathrm{oo}}\left(1 + \frac{q}{(Z\chi)}\right)\right]^{-1} = \left(1 + \frac{1}{R_2' Y_{\mathrm{o}}}\right)^{-1}. (12)$$

Analysis (12) shows that the optimum slip s_{opt} depends only on parameters of the IM (R_2', L_{oo}, *q*) and optimal operating point $(Z\chi)_{\mathrm{opt}-\eta}$ at which it $\eta = \eta_{\mathrm{P}} \cos\varphi$ will have a maximum value regardless of the load torque T_{load}. The value $\cos\varphi$ can be easily found by using the input conductance (8), since $\varphi = -\mathrm{Arg}\, Y_{\mathrm{i}}$.

For specific values of M_{load} the stator phase voltage is easily determined from the equation of balance of electrical and mechanical power at the output of the IM (the module of χ_{opt} is used):

$$U_1 = \frac{U_2'}{\chi_{\mathrm{opt}}} = \frac{1 - s_{\mathrm{opt}}}{\chi_{\mathrm{opt}}} \sqrt{\frac{2\pi f R_2'}{3 p_{\mathrm{p}} s_{\mathrm{opt}}} T_{\mathrm{load}}},$$

where p_{p} is the number of pairs of IM poles.

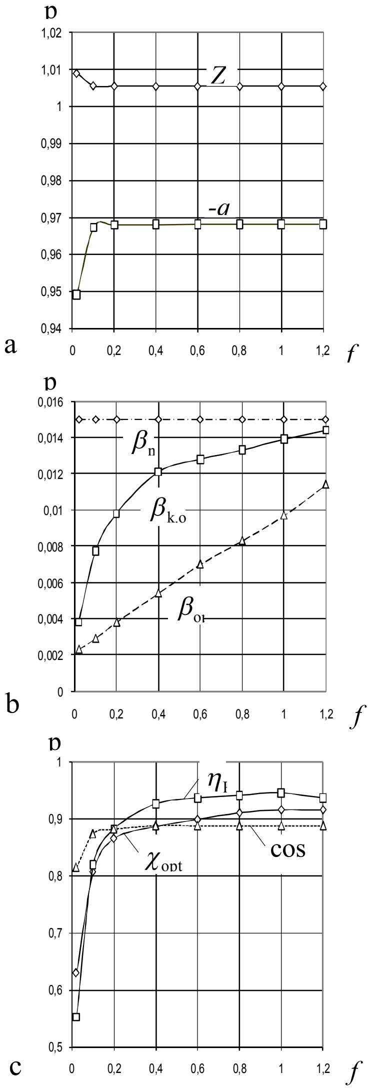

Fig.3. Results of the mathematical modeling of the frequency controlled IM

To compare the effectiveness of the proposed method of mathematical modeling with the one described above, let us take the same IM as in [2] – A-114-6M –

with the following parameters: $P_n = 320$ kW, $f_n = 50$ Hz, $U_{1n} = 380$ V, $p_p = 3$, $R_1 = 0{,}0207\ \Omega$, $R'_2 = 0{,}017\ \Omega$, $L_{\sigma1} = 0{,}551$ mH, $L'_{\sigma2} = 0{,}392$ mH, $L_m = 14{,}5$ mH, $R_m = 40\ \Omega$, $\beta_n = 0{,}015$. Parameters of the PC, which simulates the electromagnetic part of the IM, defined by (10) and (11) as a function of the relative frequency f^* of the stator voltage, are shown in Fig.3,a.

Fig.3,b would represent the dependence $\beta_{\text{k.opt}}(f^*) = s_{\text{opt}}(f^*) f^*$ calculated through (12) for optimum operating points $(Z\chi)_{\text{opt}-\eta}$ for each IM frequency f. The comparison with the direct line β_{n} shown on the same figure, which corresponds to the responsible economic frequency control of M.P. Kostenko, and the curve $\beta_{\text{opt}}(f^*)$ found in [2] as a result of minimizing the main electromagnetic losses, shows that the obtained dependence $\beta_{\text{k.opt}}(f^*)$ occupies an intermediate position between the two and can be viewed as quasi-optimal. Dependence $\chi_{\text{opt}-\eta}(f^*)$ with the calculated optimum energy parameters $\eta_{\text{P}}(f^*)$ and $\cos\varphi(f^*)$ is shown in Fig.3,c. These results are in agreement with those given in [2].

Conclusions. 1. The proposed method of energy modeling of the steady state processes is universally applicable, simple and clear.

2. Using this method for analyzing the energy patterns of the frequency controlled IM allows one to determine the parameters that are very close to optimal in the minimal basic electromagnetic losses they permit, without calculating the individual components of these losses (Fig. 3).

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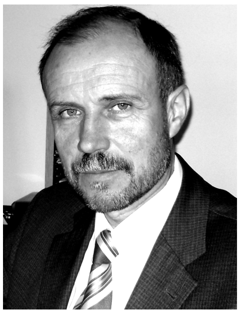
Received 14.07.2011



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