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SIMULATION MODEL OF A MISSILE STRIKE

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Abstract. *As modern naval warfare systems and tactics continue to evolve, it is crucial to employ contemporary tools capable of assessing their potential across various combat scenarios. This study model simulates interactions between anti-ship missiles and naval forces during landing operations. Our research seeks to develop a robust method for quantitatively analyzing the impact of anti-ship missiles on naval vessels under diverse combat conditions. The model's scalability allows for the incorporation of emerging naval tactics and procedures, maintaining its relevance and utility in dynamic strategic environments. It supports enhancements in precision through statistical analysis and ongoing assessments of new weaponry and technological advancements. By simulating a broad range of missile types, naval strategies, warship classes, aircraft, reconnaissance systems, and electronic warfare capabilities, the model provides insights into the strategic utility of anti-ship missiles. Our findings aim to optimize the allocation of military resources by accurately determining the necessary missile quantities to thwart enemy naval landings, thereby enhancing the effectiveness and efficiency of defensive operations.*

Keywords: *Anti-ship missiles, missile guidance, naval landing, simulation framework, missile damage estimation.*

Introduction

In the evolving landscape of modern military strategy, the interplay between artillery, anti-ship missiles and naval forces during landing operations emerges as a critical factor in ensuring coastal security. Therefore, there is an urgent need for an efficient assessment method that can effectively assess the damage effectiveness of anti-ship missiles and need in automated system for calculating the probability of missile guidance, hitting targets with missiles, determining the probability that missiles will not be shot down by enemy air defense forces, and the deployment of forces to perform the task. This is necessary to improve the effectiveness of defense missions and optimize the use of ammunition.

Our simulation framework models and analyzes a diverse array of scenarios, each varying in key tactical components. This includes an extensive range of missile types, each with distinct capabilities and specifications [1]. Additionally, we examine various launcher setups to understand how deployment strategies influence combat outcomes. The simulations also incorporate an assortment of naval vessels, taking into account different ship classes and their operational formations. We evaluate the effectiveness of these ship formations in both offensive and defensive maneuvers. To further enhance the realism of our scenarios, we integrate advanced air defense systems, air interceptors assessing their role in mitigating missile threats. [2] This approach enables us to assert the effectiveness of anti-ship missiles in a multitude of simulated combat situations, offering a detailed understanding of their strategic value in modern naval warfare.

From the perspective of ground defense operations, it is imperative to conduct highly precise calculations to determine the likelihood of successfully accomplishing the assigned objective, which is to hinder the naval landing of enemy forces. This process involves a detailed analysis of various factors [3, 4], including the assessment of enemy capabilities, the evaluation of our own defensive strengths and weaknesses, and the consideration of environmental variables that could impact the operation [5]. The objective is to understand, with as much accuracy as possible, the probability of successfully repelling an enemy naval invasion. Based on these calculations, it becomes essential to make strategic decisions regarding the allocation of resources. This entails determining the quantity and type of defensive assets required, such as manpower, artillery, anti-ship missile systems, to optimally position ourselves for achieving the set goal. The focus is on ensuring that resources are utilized efficiently and effectively, balancing the need for robust defense with the constraints of available resources.

In conducting an evaluation of data obtained from multiple recent global conflicts, it is possible to establish approximate baseline parameters for each component within our analytical model [6, 7]. This includes determining the effective range of missiles, assessing the magnitude of impact, and evaluating the maneuverability, durability, and anti-air capabilities of various ships. Additionally, the model will incorporate an analysis of EW effectiveness. These baseline values are critical for facilitating a detailed analysis and understanding of the operational capabilities and limitations within a defined context.

1. Literature sources analysis

Analyzing existing models and methods of assessment anti-ship missile effectiveness from various sources we can use them as a baseline for our research [8, 9].

Literature introduces an advanced guidance strategy for multiple anti-ship missiles to coordinate an attack on naval targets, enhancing their ability to penetrate ship defenses by synchronizing their arrival times [2, 7]. This method employs a modified version of proportional navigation guidance, adjusted for biases to manage the consensus on impact timing, while accounting for practical limitations like missile field-of-view constraints. It distinguishes itself by requiring less stringent network topology conditions, showing adaptability to both static and dynamic communication scenarios among missiles. Through simulations, it includes scenarios that test the guidance law under various conditions, such as different initial positions and velocities of missiles, changing communication network topologies among the missiles, and the presence of external disturbances like wind. These simulations demonstrate the ability of the guidance law to achieve synchronized impact times, showing how the missiles can adjust their trajectories dynamically to meet the target engagement criteria despite the complexities and uncertainties inherent in a real-world operational environment.

Literature builds mathematical models for a combatant ship in both intact and damaged conditions. It uses an Estimation-Before-Modeling approach to estimate state variables and parameters separately, analyzing hydrodynamic forces' tendencies [10, 11]. Differences between intact and damaged conditions were analyzed, and their effects on maneuverability were verified through simulation. The literature confirms ships maneuvering characteristics in damaged conditions and provides insights into ships behavior post-damage.

We can take base parameters and info regarding missiles range, damage, guidance capabilities, ships and aircraft technical characteristics and interception difficulties from the literature [12, 13, 14].

Literature presents two new guidance laws based on differential game theory for an attacker in a scenario involving an attacker, a defender, and a target. These laws aim to enable the attacker to evade the defender and maintain a critical distance from the target without needing to know the control efforts of the target and defender. The guidance laws differ based on initial parameters, with their effectiveness demonstrated through numerical simulations [9]. And provides a strategy for attackers to successfully hit defended targets in missile defense scenarios.

Literature provides tactics, and combat activity protocols. On that basis we can build a model.

We can divide the primary functions of naval landing assault operation in several groups:

- Maritime Transportation and Landing Operations: These ships are tasked with the transportation of personnel and military equipment of naval forces across sea routes. Their design enables them to execute landings on unprepared coastlines and in aquatic environments, ensuring the successful deployment of assault forces in diverse geographical settings.
- Reloading and Unloading of Landing Forces: A critical task involves the transshipment of landing troops from auxiliary ships and larger landing crafts. This process encompasses the efficient unloading of personnel and equipment onto coastal zones or water-based locations within the designated landing areas.
- Fire Support During Approach: As these ships approach the intended landing points, they are equipped to engage in offensive operations against enemy forces. This includes the utilization of onboard weapon systems for the suppression and destruction of enemy personnel and military assets, thereby facilitating a safer landing environment for the assault troops.
- Logistical Support and Medical Evacuation: In addition to combat-oriented functions, landing ships are responsible for delivering essential material resources to the forces on the ground. This role extends to the critical task of evacuating wounded and sick personnel from the frontlines, ensuring their safe transport for medical attention.

The costal guard can launch strikes during each of these phases. It is important to make the decision considering given resources, current state of the battlefield and info from other military structures, at which phase it is more effective to attack.

2. Objectives and tasks of the research

There are two parties involved in the model. The attacker, whose task is to successfully escort the landing order and carry out the landing. The attacking side consists of landing ships, guard ships, support ships and air support in the form of reconnaissance, EW systems and interceptor fighters. And the defender side which consists of missile launcher.

The main task of the defending side is to prevent landing operation by targeting and neutralizing as many of the enemy's landing ships as possible using anti-ship missiles. And it is crucial to ensure that the defending side can react with maximum efficiency to any emerging threats, by having all the required equipment and teams that are prepared and strategically positioned to execute their tasks.

The first objective of the article is to develop and improve both models and methods designed for accurately predicting and estimating damage from a missile strike to enemy naval landing ships, considering all variables listed above.

The second objective is to identify and refine a systematic approach to determine the optimal quantity of missiles required to successfully accomplish the assigned task of preventing naval landing operation.

3. Model and methods

The basis of the Methodology is mathematical modeling of the process of application of anti-ship missiles to justify the most optimal way of their application.

The Force disposition is determined based on the assigned task, which contains certain criteria for the effectiveness of its solution. To determine the required composition of forces, it is necessary, based on the tactical capabilities of missile carriers, their weapons and the conditions of the tactical situation, to determine the necessary composition of the strike forces, variants of the composition of group volleys and their sequence [12, 15]. The supply of missile launchers is affected by the composition of the defense ships because the effectiveness of solving the task depends on their combat capability. The calculation must also be carried out according to the different variants of the defense forces, which will allow assessing the impact of their quantitative and qualitative composition on the possibilities of solving the combat task.

In general, the increase in the composition of the security forces, their rational construction and use in the system of universal defense leads to a decrease in the number of missile carriers.

The required number of anti-ship missile carriers and their security forces is determined based on calculations. For the convenience of the commander's work in analyzing the results of the calculations, it is necessary to form them in a way that will show the dependence of the effectiveness of solving the combat task on the conditions adopted in each of the options. Such calculations with conclusions must be performed in advance regarding specific variants of the composition of one's own forces, the forces of the enemy, and possible conditions of the tactical situation. After receiving a combat assignment, we can use calculations made in advance and, if necessary, refine them using this methodology.

To be able to make an estimation of results of missile strike we need to make calculations for every step:

- Calculation of the probability of missile interception by enemy fighter aircraft.
- Calculation of the probability of intercepting missiles by anti-aircraft fire means of adversary ships.
- Determination of probability of aiming missiles at the targeted ships or defense ships.
- Determination of the probability of destroying target ships with one launch.

[8, 16] The probability of destroying a target depends on a number of factors, the main of which are the following:

- Type of the target(ship) and its durability.
- The number of missiles that reached the target.
- Damage potential of missile type.
- The meaning of "hitting the target" in assigned combat task which meets the performance criteria.
- Resistance from enemy anti-air means.
- Homing potential of missile targeting core.

To calculate the probability of missile interception by enemy fighter aircraft we must take into account the set of factors [17]: the number of launched missiles, total time needed to launch all missiles, the number of active enemy interceptors, supporting aiming systems, the loadout of every fighter jet and the amount of available ammo for them.

We can calculate the probability [5] of the missiles to evade enemy interceptor of k type using this formula:

$$Q_{fak}^{(j)} = 1 - P_{\mu ak}\left(1 - e^{-\frac{m\mu_k}{n}}\right), \qquad (1)$$

Where $P_{\mu ak}$ is the probability of k type fighter jet to aim at missile, which also depends on type of supporting aiming systems from, m is amount of active fighter jets,
μ_k is the damage potential of a k type fighter jet against the j type missile [18]. n is the number of missiles in the salvo.

Usually, fighter jets of each model work in separate groups so if multiple groups of enemy fighter jets are active, we can calculate the chance of missiles to avoid all groups of enemy fighter jets with the formula:

$$Q_{fat} = \prod_{k}^{N_f} Q_{fak}, \quad (2)$$

Where N_f is number of different types of fighter jets supporting naval group.

Now we can calculate an estimated number of missiles that avoided the enemy interceptors using formula:

$$\bar{n} = n\, Q_{fat}, \quad (3)$$

where n is the total number of missiles in the salvo.

Probability of intercepting missiles by anti-aircraft fire means of adversary ships depends on ships order structure, the area which they are covering and anti-air capabilities of every ship in "core" and "defense" groups and total time range needed to launch all missiles [8, 14].

To determine the time range required by each launcher to deploy all missiles [21, 27], we will use the following formula:

$$T_j = \left(n_j - 1\right)\tau_j, \quad (4)$$

where n_j the number of missiles launched from the j-th carrier and τ_j the time interval between subsequent rocket launches from the j-th carrier.

The total time span of the full salvo can be determined by the formula:

$$T_a = T_j max\left(1 + k_s\left(N_p - 1\right)\right), \quad (5)$$

where $T_j max$ is maximum value of T_j between all carriers. N_p is the total number of carriers which is active during missile attack. $k_s = 0.2$ is a statistical coefficient.

To calculate the probability and number of missiles which will avoid the enemy ships anti-air firearms we have to analyze and make numerical coefficients for anti-air potential of every enemy ship type [18].

To determine the total striking potential of anti-aircraft means for ships from core and defense groups we can use formula:

$$\mu^c_{aa} = \sum_{k=1}^{N_c} \mu^c_{aak}, \quad (6)$$

$$\mu^d_{aa} = \sum_{k=1}^{N_d} \mu^d_{aak}, \quad (7)$$

where N_c and N_d the number of ships core and defense groups, respectively. μ_{aak} is anti-air potential of k-type ship, which depends on total time range of missile attack.

We can calculate estimated probability of missiles not being shot down by ships anti-air systems for core and defense using these formulas:

$$Q_{aa(c)} = e^{-\frac{\mu_c}{n}}, \quad (8)$$

$$Q_{aa(d)} = e^{-\frac{\mu_d}{n}}, \quad (9)$$

where n is the total number of missiles in the salvo. And μ_c and μ_d - anti-air damage potential for all ships within group, which we can calculate by using formulas:

$$\mu_c = K^c \mu^c_{aa} + K^o \mu^d_{aa}, \quad (10)$$

for core group, and

$$\mu_d = K^o K^d \left(\mu^d_{aa} + \left(1 - K^c\right)\mu^c_{aa}\right), \quad (11)$$

for the defense group. Where K^c,K^d,K^o- is a coefficient of ships anti-air fire means effectiveness for core, defense groups, and whole order respectively, which is dependent on an order structure and operational radius [14, 19]. The more spread out the ships are, the less effective their anti-aircraft defenses are.

Determine P_{cap} the probability that, in conditions without electromagnetic interference countermeasures, each missile in salvo will be aimed at the ships that are designated for destroying (ships in core group). The value is chosen depending on the type of missile, [20, 21] the type of group of enemy ships, the radius at which the ships are located from the center of the order, as well as the number of core ships and defense ships.

Determine P_{ac}- the probability that in the conditions of EW, each salvo missile will be directed at the ships that are determined to be hit (main targets, ships in core group). [20, 21] Similarly, the value is chosen depending on the type of missile, the type of grouping of enemy ships, the radius on which the defense ships are located, as well as the number of core ships and defense ships.

Determine P_{ad} the probability that in the conditions of EW, each of the missiles of the salvo will be aimed at the defense ship. This corresponds to the share of missiles that will be directed at the defense ships:

$$P_{ad} = Q_{ew} - P_{ac}, \quad (12)$$

where Q_{ew} - the probability that, in the conditions of EW, each of the missiles of the salvo will not be deflected and will be aimed at the core ship or at the defense ship (coefficient that defines the number of missiles that are not affected by the impact of the enemy's EW).

If ships order contains the ships of same type, in that case we can consider all of them as targets in core group and P_{ac} taken as equal to Q_{ew}.

The probability of capture by a cruise missile at the target is indicated in the firing rules of the anti-ship defense complex in accordance with the conditions of the situation with a level of detail that allows a decision to be made regarding the aiming point of individual or all salvo missiles [22, 23]. Such conditions are lining up the order of enemy ships, the direction of the strike, the presence of false targets alongside the real ones, the location of the object of the strike near the coast (which can lead to the capture by the missile of the objects on the coast), etc.

Mathematical expectation of destroying targeted i-th ship in order of N ships with one strike can be calculated with the formula:

$$W_{i(n)} = 1 - e^{-\left(\frac{nP_{cap}P_{hom}Q_{aam}(n)Q_{ew}Q_{tech}}{\omega_i}\right)}, \quad (13)$$

for every ship in core and defense groups, where

P_{hom} - coefficient which determines the probability of a missile hitting a target captured by it with self-homing, it depends on a missile type and ship size [24].

Q_{ew} - the probability of aiming the missile at the target in the conditions of EW.

P_{cap} - the probability that the missile will capture and target the i-th ship from core or defense group [15, 25].

Under the condition of firing at a compact group of warships from N ships and aiming at the center of the order, missiles will be evenly distributed among the group's targets. Under such conditions, the probability of targeting a ship in the core group can be calculated using a simplified formula $\frac{n}{N}$, where n is the number of missiles in the salvo, and N is the total number of ships in the group.

$$Q_{aam(n)} = Q_{fa} Q_{aa(n)}, \quad (14)$$

Q_{aam} - the probability that the salvo will not be hit by anti-aircraft means of the enemy, which is calculated separately for each ship in core or defense groups.

Q_{tech} - the probability that the rocket will not fail due to technical reasons.

ω - the average required number of missile hits to destroy the target.

The mathematical expectation of the number of affected ships from the group can be determined by the formula:

$$\overline{N} = \sum_{i=1}^{N} W_i, \quad (15)$$

Determination of the degree of completion of the combat task with the formula:

$$W_{tc} = \frac{\overline{N_k}}{N_k}, \quad (16)$$

Where $\overline{N_k}$ is mathematical expectation of target hits, and N is total number of targets in core group.

All coefficients for different ship properties, such as anti-air fire potential, ships distribution inside groups, fighter jets ammo and fire potential, missiles homing efficiency, effective range and damage estimation gathered and calculated from observing and analyzing many real operations and confrontations around the world [11]. To test and analyze our model in different setups we will use generalized and simplified values for all coefficients.

We will consider the general case when one hit is enough to disable a ship. In that case we can simplify the task of calculating results of multiple strikes in our model. Using the results of each strike we can update the status of each ship depending on total math expectation for targets that were hit. Because 1 hit is enough to disable or destroy a ship, then we can just remove every ship that was hit from our next calculations.

The main objective is to determine the composition of launcher groups and total number missiles in the salvo needed to be able fulfill assigned degree of task completion W_{tc}. In further calculations we will set $W_{tc} = 1$, which mean destruction of every ship in the core (naval landing) group.

If the general mathematical expectation of the number of targets hit is set as a criterion, the required force must be capable of hitting such a mathematical expectation of targets N:

$$\overline{N} = N - \frac{N - \overline{N_{tl}}}{\left(1 - W_{sl}\right)\left(1 - W_{\Sigma}\right)}, \quad (17)$$

where N - the total number of targets in the group that must be hit and N_{tl} - the total given mathematical expectation of the number of targets hit.

In addition to the calculations for evaluating the effectiveness of the combat task, which are performed to determine the composition of the forces, it is necessary to conduct calculations for evaluating the effectiveness of strikes. They are carried out under the condition that a certain number of anti-ship missile carriers in the strike groups will remain operational by the time of the strikes. Such calculations are necessary to clarify the organization of strikes in conditions where the actual composition of forces will change due to possible losses of carriers during deployment and during combat.

[4] The damage potential of an anti-ship missile in relation to a specific target is determined by the conditional law of target hit, which establishes a functional relationship between the survivability of the target ship, the characteristics of the anti-ship missile that hits it, and the number of missiles that hit the target and led to the withdrawal, target failure or its destruction (sinking). In some cases, one anti-ship missile hit is enough to destroy a target.

$$W(n) = \sum_{m=0}^{n} P_{n,m} G(m) = 1 - \left(1 - \frac{p}{\omega}\right)^n$$

This formula allows us to generally estimate the effectiveness of hitting a ship target based on the average number of anti-ship missiles hitting the target ω and the probability of the missile hitting the target p.

Where n - number of missiles in the salvo, m - number of missiles that hit the target. $P_{n,m}$ - the probability that m missiles from a total of n missiles hit the target. p - the probability of the missile hitting the target.

4. Experiment

Let's compare results for different setups naval landing group, defense ships group, air support and different setups of missile launchers.

1) The enemy ships order consists only of 4 ships of "Type-A1" from naval landing group (core group). The coastal guard side has 2 launchers with 4 missiles of the same type each. Naval landing group don't have air support and there is no EW active.

In all calculations we will use one type of anti-ship missile "Type-M1".

Because there are no supporting enemy fighter jets and there is no enemy EW:

$Q_{fa} = 1$, and $Q_{ew} = 1$

To be able calculate ships anti-air damage potential we also need calculate time range needed to launch all missiles using formulas (4) and (5). We have 2 launcher2 and each one of them contains 4 missiles. Time between strikes $\tau = 5$ s. Time for each launcher $T_j = 15$.

Because booth launchers are identical:

$T_j max = 15$

Then we can calculate total needed time:

$T_a = 15(1 + 0.2(2 - 1)) = 18.$

For ships of "Type-A1" and given time we can set the anti-air damage potential coefficient $\mu_{aa} = 0.2$. Because in our model "Type-A1" ships are for naval landing, they have relatively small anti-air capabilities.

Total value for core group $\mu_{aa}^c = 4 \cdot 0.2 = 0.8$ and for defense group $\mu_{aa}^d = 0$. Based on a radius of naval group order $R = 5$ (km) and ships number we can get coefficient of total anti-air efficiency for core group $K^c = 0.55$.

And we can get total anti-air damage potential for ships in core group:

$\mu_c = 0.55 \cdot 0.8 = 0.44,$

we can get a probability of missiles to evade enemy ships anti-air:

$$Q_{aa(c)} = e^{-\frac{0.44}{8}} \approx 0.9465.$$

Because there are 4 identical ships in a core group: $P_{cap} = \frac{1}{4}$,

And for ship "Type-A1": $P_{hom} = 0.9$.

$Q_{tech} = 0.95$, and $\omega = 1$.

After we can calculate mathematical expectation of destroying for each ship in core group with (13) formula:

$$W_i = 1 - e^{-\left(\frac{8 \cdot 0.25 \cdot 0.9 \cdot 0.9465 \cdot 1 \cdot 0.95}{1}\right)} \approx 0.802$$

Total math expectation because we have 4 ships of same type $\overline{N} = 4 \cdot 0.802 = 3.2$, which mean that with 2 launchers and 8 missiles total, there is a good probability of destroying 3 out of 4 ships in naval landing(core) group.

We can calculate the probability of task completion with different numbers of prepared missiles.

Based on a graph with the results we can see that to be able to destroy all 4 ships from the core group we need 15 missiles and 4 launchers to get total math expectation $\overline{N} > 3.8$ ($\geq 95\%$)

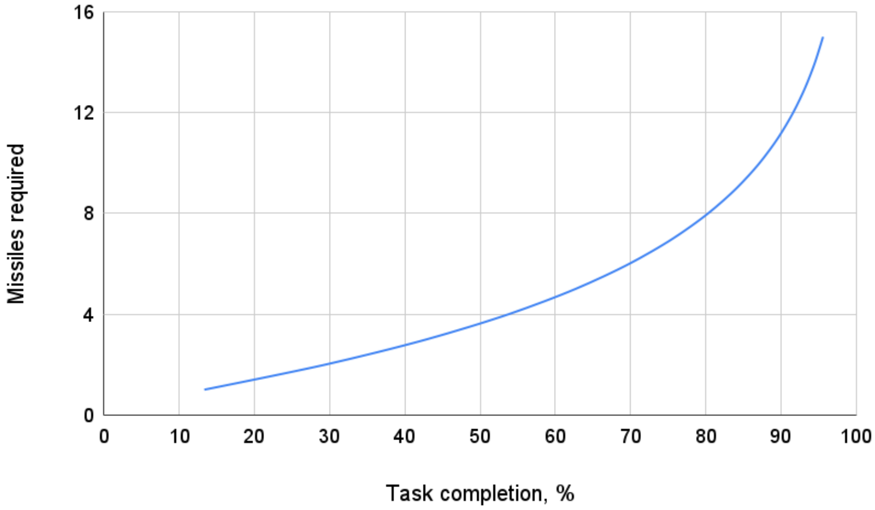

Fig. 1. Missiles required against enemy order with 4 naval landing ships.

2) Now we will add 4 ships of "Type-D1" to the defense group and estimate how they affect our damage assessment to the ships in naval landing (core) group. And how many launchers and missiles we must add to coast guard forces to be able successfully complete the task of destroying all naval landing ships.

There are no supporting enemy fighter jets and there is no enemy EW so $Q_{fa} = 1$, and $Q_{ew} = 1$.

Time range for missile strike is the same $T_a = 18$.

For ship of type "Type-A1" the anti-air damage potential coefficient $\mu_{aa} = 0.2$. And for ships of "Type-D1" and given time we can set the anti-air damage potential coefficient $\mu_{aa} = 2$, because this ship is from defense group it does has relatively strong anti-air capabilities. Total value for core group $\mu^c_{aa} = 0.8$ and for defense group $\mu^d_{aa} = 4 \cdot 2 = 8$.

Based on a radius of naval group order $R = 5$(km) and ships number we can get coefficient of total anti-air efficiency for core group $K^c = 0.55$, for defense group $K^d = 0.55$ and for the whole order $K^o = 0.4$.

And we can get total anti-air damage potential for ships in core group with formula (10):

$$\mu_c = 0.55 \cdot 0.8 + 0.4 \cdot 8 = 3.64,$$

and for defense group with formula (11):

$$\mu_d = 0.4 \cdot 0.55(8 + (1 - 0.55) \cdot 0.8) \approx 1.84$$

We can get a probability of missiles to evade ship anti-air for each ship in core group:

$$Q_{aa(c)} = e^{-\frac{3.64}{8}} \approx 0.634,$$

and for each ship in defense group:

$$Q_{aa(d)} = e^{-\frac{1.84}{8}} \approx 0.795$$

Now because there are two separate groups of ships P_{cap} is different for each of them. For ships in core group: $P_{cap} = \frac{0.6}{4} = 0.15$, and for the ships from defense group: $P_{cap} = \frac{0.36}{4} = 0.09$.

Coefficient for probability of a missile hitting a captured target for ship "Type-A1" $P_{hom} = 0.9$.

And for ship "Type-D1" $P_{hom} = 0.7$, because it is a smaller ship class.

$Q_{tech} = 0.95$, and $\omega = 1$.

Now we can calculate mathematical expectation for hitting ships in core group with formula (13):

$$W_{i(c)} = 1 - e^{-\left(\frac{8 \cdot 0.15 \cdot 0.9 \cdot 0.634 \cdot 1 \cdot 0.95}{1}\right)} \approx 0.478,$$

and for ships in defense group:

$$W_{i(d)} = 1 - e^{-\left(\frac{8 \cdot 0.09 \cdot 0.7 \cdot 0.795 \cdot 1 \cdot 0.95}{1}\right)} \approx 0.316$$

Total math estimation for core group $\overline{N}_c = 4 \cdot 0.478 = 1.912$,

And for defense group:

$$\overline{N}_d = 4 \cdot 0.316 = 1.264,$$

And total damage estimation for full order:

$$\overline{N} = 1.912 + 1.264 = 3.176.$$

Comparing the two results we can see how much impact on a naval landing operation adds defense group of ships.

While the total expected number of destroyed ships still equals ~3.2, our main goal is to destroy all 4 ships of core (naval landing) group. But with 2 launchers and 8 missiles we can reliably expect to hit only 2 ships out of 4.

Using the same method as in our first example we can start adding new launchers with additional missiles until we have a reliable chance to destroy all 4 ships in core group.

After a few iterations we will find out that we need 7 launchers with a total 27 missiles to get: $\overline{N}_c \approx 3.805$.

Analyzing this example further we can see that one of the factors that decreased effectiveness of missile strike was additional anti-air tools from ships in defense group. But the main factor was adding new targets for automatic homing systems of missiles of chosen type. Adding new targets drastically decreases chances for homing systems to identify targeted ships from naval landing (core) group.

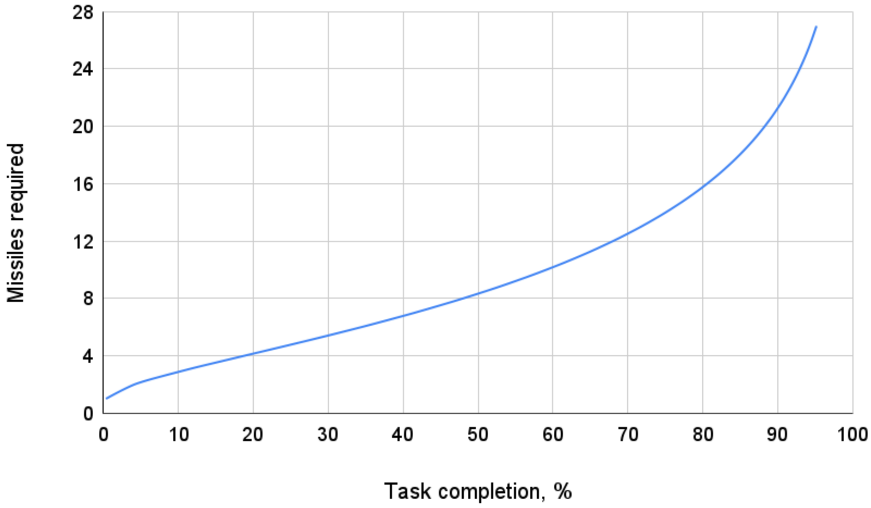

Fig. 2. Missiles required against enemy order with 4 additional defense ships.

And to be able successfully complete the assigned task of destroying all ships in the naval landing group with probability $>95\%$ we need almost twice as many resources, from 4 launchers and 15 missiles to 7 launchers and 27 missiles.

3) Next calculation will have same setup of ships in core and defense groups, and same number of launchers and missiles but also will include air support in a form of 2 fighter jets of type "Type-FJ1".

Because there is no active enemy EW, coefficient will be equal: $Q_{ew} = 1$.

For fighter jets with "Type-FJ1" we need values for a coefficient that defines probability of fighter jets to aim missiles, which is dependent on a assisting aiming systems, we will use generalized value: $P_{\mu ak} = 0.75$.

And a coefficient that defines overall damage capabilities of this type of jets [26] $\mu_k = 2.5$.

Now we need calculate probability for missiles to evade enemy interceptors with formula (1):

$$Q_{fa} = 1 - 0.7 \cdot \left(1 - e^{-\frac{2 \cdot 2.5}{8}}\right) \approx 0.675,$$

which means that we can expect ~5.4 missiles to avoid fighter jets and reach ships order.

Time range for missile strike is the same $T_a = 18$.

Anti-air capabilities of each group of ships have the same values:

$\mu_c = 3.64$ and $\mu_d = 1.84$.

And we can get a probability of missiles to evade ship anti-air for each ship in core group:

$$Q_{aa(c)} = e^{-\frac{3.64}{5.4}} \approx 0.509,$$

and for each ship in defense group:

$$Q_{aa(d)} = e^{-\frac{1.84}{5.4}} \approx 0.711$$

Coefficients for target capturing are the same,
$P_{cap(c)} = 0.15$ for core group, and $P_{cap(d)} = 0.09$ for defense group.
And $P_{hom(c)} = 0.9$ for ships in core group, and $P_{hom(d)} = 0.7$ for ships in defense group.

$Q_{tech} = 0.95$, and $\omega = 1$.

Now let's calculate mathematical expectation for hitting ships in core group:

$$W_{i(c)} = 1 - e^{-\left(\frac{8\cdot 0.15\cdot 0.9\cdot 0.509\cdot 0.675\cdot 0.95}{1}\right)} \approx 0.297,$$

For probability to hit a ship from defense group:

$$W_{i(d)} = 1 - e^{-\left(\frac{8\cdot 0.09\cdot 0.7\cdot 0.711\cdot 0.675\cdot 0.95}{1}\right)} \approx 0.205$$

Total math expectation for core group:

$\overline{N}_c = 4 \cdot 0.297 = 1.188,$

And for defense group:

$\overline{N}_d = 4 \cdot 0.205 = 0.82,$

And total damage estimation for full order:

$\overline{N} = 1.188 + 0.82 = 2.008.$

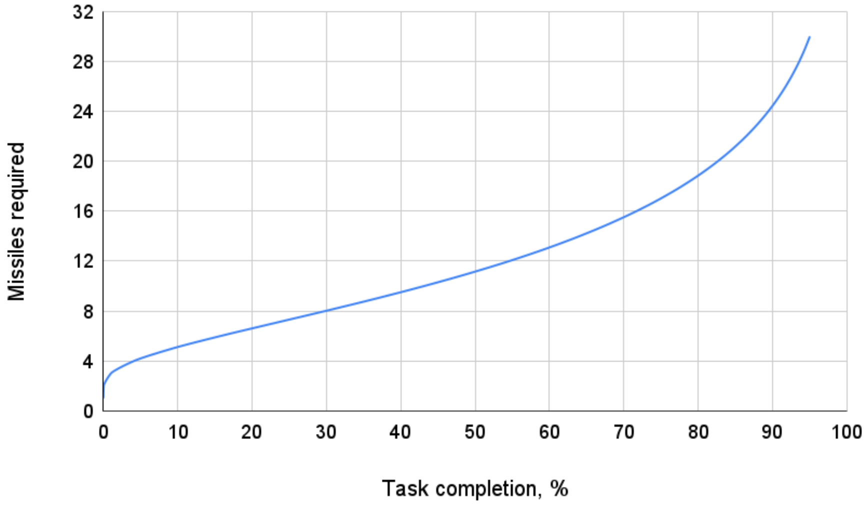

Fig. 3. Missiles required with enemy air support.

And to be able destroy all 4 ships with probability $> 95\%$ from core group we will find out that we need 8 launchers 30 missiles.

Analyzing current setup with air support we can see that fighter jets are very effective against a small number of missiles, but because of the limited amount of ammo they are much less effective against a bigger number of missiles. So based on the result, if there are enemy air-support expected, the commander should prepare 2 additional missiles for every fighter jet of "Type-FJ1".

4) Next calculation will also include active enemy EW. $Q_{ew} = 0.5$.

Based on the type of grouping of enemy ships, the radius of order and number of ships in core group, EW coefficient for core group:
$P_{ac} = 0.3,$
And for defense group:

$P_{dc} = 0.5 - 0.3 = 0.2.$

All other parameters will be the same and we calculate mathematical expectation for hitting ships in core group:

$$W_{i(c)} = 1 - e^{-\left(\frac{8\cdot 0.15\cdot 0.9\cdot 0.509\cdot 0.675\cdot 0.3\cdot 0.95}{1}\right)} \approx 0.1,$$

And for defense group:

$$W_{i(d)} = 1 - e^{-\left(\frac{8\cdot 0.09\cdot 0.7\cdot 0.711\cdot 0.675\cdot 0.2\cdot 0.95}{1}\right)} \approx 0.045$$

Total math expectation for the core group:

$\overline{N}_c = 4 \cdot 0.1 = 0.4,$

For defense group:

$\overline{N}_d = 4 \cdot 0.045 = 0.18,$

Total damage estimation for full order:

$\overline{N} = 0.4 + 0.18 = 0.58.$

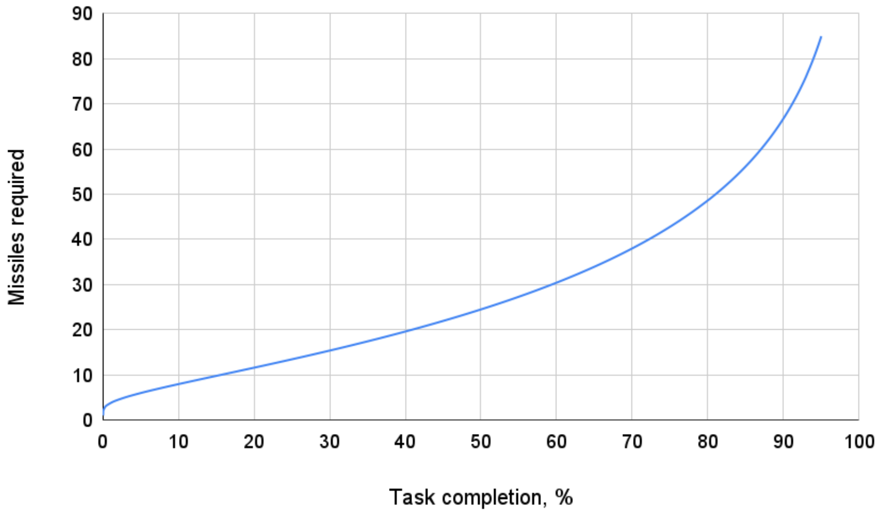

Fig. 4. Missiles required with active enemy EW.

To get the probability $>95\%$ of destroying all 4 ships in the core group we need 85 missiles and ~22 launchers.

Results show that EW interference is much more effective against missile strikes with auto homing, because EW greatly decreases capabilities of a missile to capture a designated target. [28] And it's possible that spreading resources into multiple strikes with lesser number of missiles will be more effective against such composition of enemy forces.

Comparing all the results together we can see a much clearer picture of what to expect against the different composition of enemy forces. Such calculations allow coastal guard commanders to make better decisions in a shorter period of time to be able to make all the necessary preparations against an enemy attack.

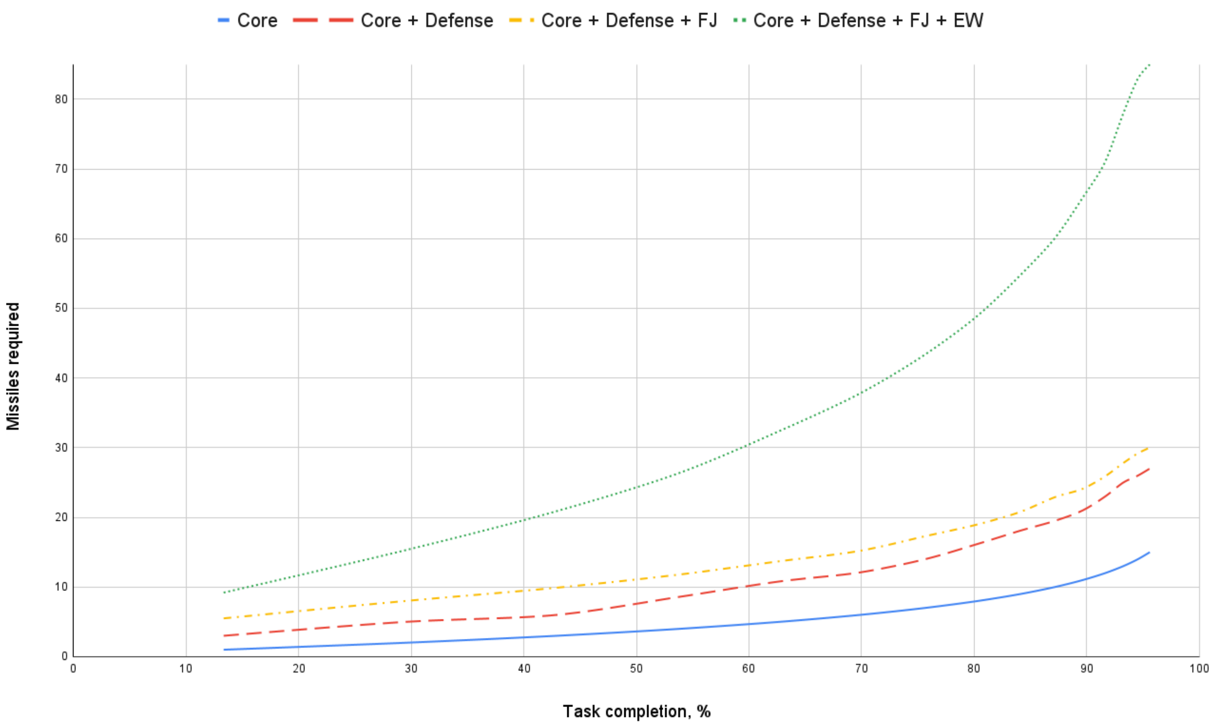

Fig. 5. Comparison for different setups of enemy landing operation.

Conclusions

A model has been constructed which allows us to make an estimation of potential missile strike damage to the ships that are part of naval landing operations.

Builded model takes into account all potential participants of naval landing operation, landing ships, defense and supply ships with different classes, air support, fighter aircrafts, different assisting recon systems and possible anti-air and anti-missile means.

The model is designed to estimate the potential damage from a missile strike against multiple possible setups of enemy forces.

The second objective is to determine the optimal number of resources required to successfully complete the assigned mission of destroying a specified number of enemy landing ships. Using the current model allows us to compare possible results with different setups and choose the optimal one in current tactical situation.

The results of experimental calculations show us an effectiveness and many possible use cases for the developed model and methods. Analyzing calculations with different setups for enemy forces we can clearly see the possible impact of every participant on the success or failure of naval landing operation, such as different types of air support, additional escorting ships with anti-air and EW systems.

Such tool gives commanders of coastal guard opportunity to make fast and effective decisions for planning against enemy activities and allows to relatively accurately predict possible outcomes and make all necessary preparations regarding size of silo, the timings, number of possible strikes and size of operational team for countering enemy naval landing operation.

Analyzing information from different world conflicts and specifications of equipment and weapons from different sides, we can further improve precision of calculations within a model. Model structure allows for further improvements. We can add new variables depending on changes in modern naval warfare or added new technologies.

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ІМІТАЦІЙНА МОДЕЛЬ РАКЕТНОГО УДАРУ

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Анотація. *Сучасні військово-морські системи і тактика стають все більш складними. Необхідно мати сучасні засоби, які можуть оцінити ефективність кожної системи в усіх можливих бойових сценаріях. У цьому дослідженні представлена комплексна імітаційна модель, призначена для оцінки стратегічної взаємодії між протикорабельними ракетами берегової охорони та військово-морськими силами в сучасних військових десантних операціях. Це дослідження спрямоване на розробку ефективного методу оцінки, який аналізує ефективність ураження протикорабельних ракет проти кораблів, які беруть участь у морських десантних операціях за різними бойовими сценаріями. Модель є масштабованою та може бути оновлена для включення нових військово-морських тактик і процедур в міру їх виникнення, гарантуючи, що модель залишається актуальною та ефективною в сучасних стратегічних умовах. Модель дозволяє додатково підвищувати точність на основі статистичних результатів і подальшого аналізу технічних характеристик нової зброї та технологій. Включаючи широкий спектр типів протикорабельних ракет, стратегій морського десантування, класів бойових кораблів, типів літаків, систем розвідки, різних засобів РЕБ і вдосконалених протиповітряних і протиракетних систем, модель імітує наступальні та оборонні маневри для визначення стратегічного значення протикорабельних ракет у морських бойових діях. Завдяки детальному аналізу дослідження спрямоване на те, щоб зрозуміти оптимальний розподіл військових ресурсів, включаючи точний розрахунок кількості ракет, необхідних для запобігання висадці військово-морського десанту противника. По-перше, дослідження спрямоване на вдосконалення моделей прогнозування та методів оцінки шкоди від ракетного удару по військово-морським силам противника. По-друге, удосконалити стратегії визначення необхідного інвентарю ракет, щоб запобігти висадці противника, забезпечуючи тим самим максимальну оперативну ефективність і результативність у оборонних місіях. Результати пропонують системний підхід до розподілу військових ресурсів і стратегічного планування в контексті запобігання морській десантній операції.*
Ключові слова: *протикорабельні ракети, наведення ракети, морський десант, структурне моделювання, оцінка розміру ураження ракети.*

Abstract. *Modern naval warfare systems and tactics becomes more and more complex. It is essential to have modern tools which can evaluate the effectiveness of each system in all possible combat scenarios. This study presents a comprehensive simulation model designed to evaluate the strategic interaction between coastal guard anti-ship missiles and naval forces in modern military landing operations. This research aims to develop an efficient assessment method that quantitatively analyzes the damage effectiveness of anti-ship missiles against ships which participate in naval landing operations under varying combat scenarios. The model is scalable and can be updated to incorporate new naval tactics and procedures as they emerge, ensuring the model remains relevant and effective in modern evolving strategic conditions. The model allows further improvements in accuracy based on statistical results and further analysis of technical characteristics of new weaponry and technologies. Incorporating a wide spectrum of anti-ship missile types, naval landing strategies, warship classes, aircraft types, recon systems, different means of EW, and advanced anti-air and anti-missile systems, the model simulates offensive and defensive maneuvers to determine the strategic value of anti-ship missiles in naval warfare. Through detailed analysis, the study seeks to provide insights into the optimal allocation of military resources, including the precise calculation of missile quantities needed to prevent enemy naval landings operation. Firstly, the research aims to improve prediction models and methods for estimating missile strike damage on enemy naval forces. Secondly, to refine strategies for determining the necessary missile inventory to prevent enemy landings, thereby ensuring maximum operational efficiency and effectiveness in defense missions. The results offer a systematic approach to military resource allocation and strategic planning in the context of preventing naval landing operation.*
Keywords: *Anti-ship missiles, missile guidance, naval landing, simulation framework, missile damage estimation.*

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