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SUBOPTIMAL NONLINEAR CONTROL OF ARMATURE CURRENT LOOP AT DIRECT TORQUE CONTROL

Abstract. *Synthesis of nonlinear suboptimal regulator, which provides high quality of DC motor torque control, is performed. Investigation of system with regulator synthesized using numerical modeling method is made; an analysis of obtained results is performed.*

Keywords: *DC drive, torque control, the method of Bellman – Lyapunov, invariant embedding, unwinding drive, tension control, paper rewinding, armature current stabilization*

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СУБОПТИМАЛЬНОЕ НЕЛИНЕЙНОЕ УПРАВЛЕНИЕ КОНТУРОМ ТОКА ЯКОРЯ ПРИ ПРЯМОМ УПРАВЛЕНИИ МОМЕНТОМ

Аннотация. *Выполнена процедура синтеза нелинейного субоптимального регулятора, обеспечивающего высокое качество регулирования момента двигателя постоянного тока. Осуществлено исследование системы с синтезированным регулятором методом цифрового моделирования, произведен анализ полученных результатов.*

Ключевые слова: *привод постоянного тока, регулирование момента, метод Беллмана- Ляпунова, инвариантное вложение, электропривод размотки, регулирование натяжения, перемотка бумаги, стабилизация тока якоря*

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СУБОПТИМАЛЬНЕ КЕРУВАННЯ КОНТУРОМ СТРУМУ ЯКОРЯ ПРИ ПРЯМОМУ УПРАВЛІННІ МОМЕНТОМ

Анотація. *Виконано синтез нелінійного субоптимального регулятора, що забезпечує високі показники якості регулювання моменту двигуна постійного струму. Здійснено дослідження системи з синтезованим регулятором методом цифрового моделювання, проведено аналіз отриманих результатів.*

Ключові слова: *привод постійного струму, регулювання моменту, метод Беллмана-Ляпунова, інваріантне вкладення, електропривод розмотування, регулювання натяжіння, перемотування бумаги, стабілізація струму якоря*

Introduction. In recent years more and more important is the increasing of productivity of printing presses. One of the most important elements of the drive units is “feeder” or unwinding mechanism, providing stabilization of the torque, and therefore the tension of unwound paper [1].

There are four basic approaches to the regulation of tension in printing machines:

– torque sensorless control;

– control of tension force;

– motor speed control with dancer feedback;

– winding systems with line speed control and diameter roll feedback.

In drives with high requirements for roll winding density under wide range of the radius changing of winding material regulation system with control of linear velocity and roll diameter feedback is used. This approach eliminates the effect of the "telescope" (extrusion of the inner layers roll outward), provides high accuracy of

line speed for colored printing. Despite this, in the majority of printing machines, this approach is not used because of higher cost.

In case of machines where the effect of the "telescope" is not critical using of tension control or speed with dancer (potentiometer) feedback control system is appropriate. The disadvantage of this approach is the inability to work in contact winding machines, as well as a significant complication of mechanical parts because of using of an external torque meters.

Therefore, under the general trend of winding machines mechanics simplifying and using of complex variable speed drives, regulation systems of motor torque without tension feedback are used. They are almost indispensable in contact winding machines, easy to setup and operation. Also, these systems are used in unwinding mechanisms drives, especially in the case of low-density material rewinding.

At the same time, the accuracy of tension stabilization for this approach depends entirely from the quality of motor torque control. Therefore, in such systems DC machines, in which the torque is proportional to the armature current, are still very widely used [2, 3]. Note that, in the contact winders winding radius is small, so the excitation control is not in use and the magnetic flux of motor is constant.

Winding quality of paper is determined by the minimum response time of the system to disturbances, so current control loop maximum performance is mostly needed. It should be noted that this is not taken into account the requirements for dynamic precision, which will adversely affect the integrity of the winding material in high and ultra-high speeds processing. For maximum performance and static accuracy nonlinear circuit element like "relay characteristic with hysteresis" is intro–duced [4, 5].

The problem of such approach is the occurrence of oscillations in the current loop, which provides oscillations of motor torque and bad quality of rewinding, even a loss of paper. So, the aim of paper is torque regulator synthesis, which ensure of absolute stability of system.

Selection of regulator synthesis procedure. The block diagram of the control circuit current takes the form shown in Fig.1:

There are next designations at fig.1: U_{cs} – armature current setpoint; U_{cf} – current feedback signal; K_c, T_c, E_c – gain, time constant and output voltage of thyristor converter; R_e – equivalent resistance of the armature; T_e – time constant of armature; CR – current regulator; u_{cr} – output voltage of current regulator; I_a –armature current.

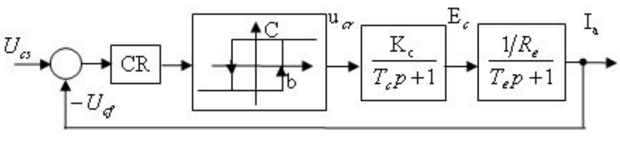

Fig. 1. Mathematical model of the loop current with switching element with hysteresis

It should be noted that, since the stabilization of tension for winding machines carried by the armature current is close to the nominal, the model does not take into account the work of the converter in discontinuous currents mode, pulsating behaviour of the inverter output voltage, and the impact of dynamic components of current during acceleration and braking motor .

There are many approaches to solve the problem of synthesis of regulators to ensure absolute stability of the highly nonlinear system.

One promising area is the use of analytical design of suboptimal controllers by Bellman – Lyapunov method applying the concept of invariant embedding [5]. This method allows us to solve the problem of synthesis of high-dimensional objects, but is suitable only for systems with univocal nonlinearities.

Thus, to solve the problem of synthesis of the regulator with the above method, "relay with hysteresis" characteristics should be replaced with model containing only univocal nonlinearity.

"Relay with hysteresis characteristics" in the following expression is described:

$$f(\sigma) = \begin{cases} C \cdot sign(\sigma - b), \dot{\sigma} < 0; \\ C \cdot sign(\sigma + b), \dot{\sigma} > 0. \end{cases} \quad (1)$$

It may be noted that, depending on (1) the sign of the derivative, sign of the hysteresis is changed only, and it can be accounted by the function. The expression (1) becomes:

$$f(\sigma) = C \cdot sign[\sigma + bsign(\dot{\sigma})]. \quad (2)$$

The block diagram of the current control circuit with (2) to the form shown in Fig. 2 is transformed.

The transition from the mathematical model of the current loop with hysteresis characteristics to a model with univocal nonlinearities can be used for the synthesis of suboptimal control law by method of dynamic programming using the concept of invariant embedding [6, 7].

Synthesis procedure of current regulator. Mathematical model of the armature current loop as a system of nonlinear differential equations is:

$$\begin{aligned} \dot{I}_a &= -\mathrm{T}_e^{-1} \cdot I_a + (\mathrm{R}_e \cdot T_e)^{-1} \cdot E_c; \\ \dot{\mathrm{E}}_c &= -\mathrm{T}_c^{-1} \cdot E_c + \mathrm{K}_c \cdot \mathrm{T}_e^{-1} \cdot C \cdot \\ &\cdot sign[\sigma + bsign(\dot{\sigma})]. \end{aligned} \quad (3)$$

To solve the problem of analytical design of suboptimal controller (ADSC) we use the procedure of "instantaneous" linearization by method of "secants".

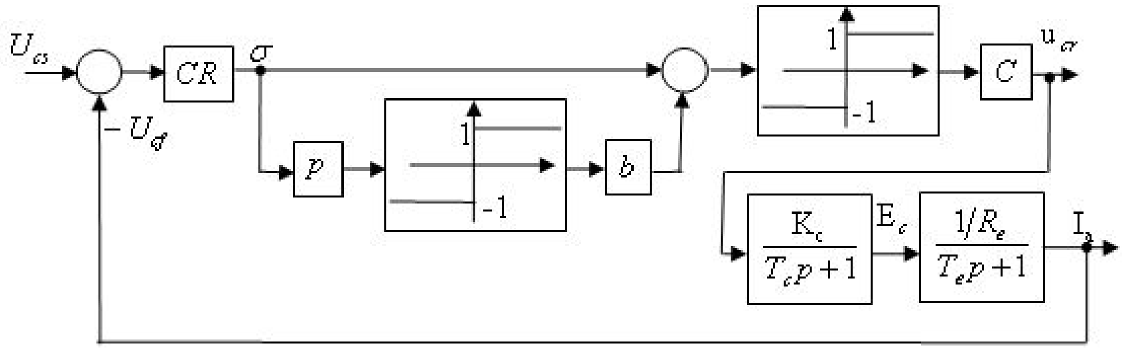

Fig. 2. Mathematical model of the current loop with univocal nonlinearities in the circuit

So, the univocal nonlinear function is replaced with a variable gain line. System of linearized differential equations describing the dynamics of the loop current, takes the form:

$$\dot I_a = -T_e^{-1} \cdot I_a + (R_e \cdot T_e)^{-1} \cdot E_c;$$
$$\dot E_c = -T_c^{-1} \cdot E_c + K_c \cdot Te_э^{-1} \cdot C \cdot K_1(\sigma) \cdot \qquad (4)$$
$$\cdot \sigma + K_c \cdot T_e^{-1} \cdot C \cdot K_1(\sigma) \cdot bK_2(\dot\sigma) \cdot \dot\sigma,$$

there $K_1(\sigma)$, $K_2(\dot\sigma)$ – coefficients of the "instantaneous" linearization of relay characteristic with two positions.

The second equation contains the derivative of the control action, to avoid we introduce an additional variable, and it will be considered as a new control action.

In this case σ as new state variable is taken. Thus, the system of linearized differential equations (4) in the normal Cauchy form is rewritten:

$$\dot x_1 = -a_1 \cdot x_1 + a_2 \cdot x_2;$$
$$\dot x_2 = -a_3 \cdot x_2 + b_1 \cdot x_3 + b_2 \cdot \sigma'; \qquad (5)$$
$$\dot x_3 = \sigma',$$

there $x_1 = I_a$; $x_2 = E_c$; $x_3 = \sigma$;
$a_1 = T_e^{-1}$; $a_2 = (R_e \cdot T_e)^{-1}$; $a_3 = T_c^{-1}$;
$b_1 = K_c \cdot T_e^{-1} \cdot C \cdot K_1(\sigma)$;
$b_2 = K_c \cdot T_e^{-1} \cdot C \cdot K_1(\sigma) \cdot bK_2(\dot\sigma)$.

Let make ADSC procedure using invariant embedding conception, consistently in few stages.

At the first step we find solution of AD for linearized system (5) and quadratic functional "in small", that is at $K_1(\sigma) = K_{11}$ and $K_2(\dot\sigma) = K_{21}$. In this case, we arrive to solution of the matrix Riccati equation in form $Q + K \cdot A + A^T \cdot K - K \cdot B \cdot R^{-1} \cdot B^T \cdot K = 0$ and find coefficients of K matrix. Control equation at solving task "at small" takes a form $\Sigma_1' = -K_1 \cdot X$, there K_1 – optimal gain matrix.

Second step. We find the solution for linearized system (5) and quadratic functional "in large" at $K_1(\sigma) = K_{12}$ and $K_2(\dot\sigma) = K_{22}$, and besides $K_{11} > K_{12}$ and $K_{21} > K_{22}$. Control equation at solving task "at large" takes a form $\Sigma_2' = -K_2 \cdot X$.

Third step. New admissible controls are defined and stinting of "instantaneous" control actions σ_1' and σ_2', correct for different regions of phase space, is made. Regulator coefficients k_1, k_2, k_3 are functions of state variables, so let suppose that variation of parameters $\Delta k_1, \Delta k_2, \Delta k_3$ are new control actions. Therefore, control law "in the large" becomes:

$$\sigma_2' = -(\sum_{i=1}^{3} k_i x_i + \sum_{j=1}^{3} \Delta k_i x_i). \qquad (6)$$

Nonclassical minimized functional (generalized criterion of Krasowski AA), meets the requirements of the dynamic accuracy and minimum costs for control, is [8]:

$$\min_{\Delta k_1,\ldots,\Delta k_3} J_3 = \int_0^\infty \left[\sum_{i=1}^{3} \alpha_i x_i^2 + \sum_{j=1}^{3} c_j \Delta k_j^2 + \sum_{j=1}^{3} c_j \Delta k_{jopt}^2 \right] dt. \qquad (7)$$

$$\dot x_1 = -a_1 x_1 + a_2 x_2;$$
$$\dot x_2 = -a_3 \cdot x_2 + b_1 \cdot x_3 - b_2 \cdot$$
$$\cdot (\sum_{i=1}^{3} k_i x_i + \sum_{j=1}^{3} \Delta k_i x_i); \qquad (8)$$
$$\dot x_3 = -(\sum_{i=1}^{3} k_i x_i + \sum_{j=1}^{3} \Delta k_i x_i),$$

For the system (7) and the minimized functional (8) Bellman equation functional has form:

$$\min_{\Delta k_1,\dots,\Delta k_3}[\ \alpha_1 x_1^2+\alpha_2 x_2^2+\alpha_3 x_3^2+c_1\Delta k_1^2+ \\ +c_2\Delta k_2^2+c_3\Delta k_3^2+\sum_{j=1}^{3}c_j\Delta k_{jopt}^2+ \\ +\frac{\partial V}{\partial x_1}\cdot(-a_1x_1+a_2x_2)+\frac{\partial V}{\partial x_2}\cdot(-a_3\cdot x_2+ \\ +b_1\cdot x_3-b_2\cdot\sum_{i=1}^{3}k_ix_i)-\frac{\partial V}{\partial x_3}\cdot\sum_{i=1}^{3}k_ix_i- \\ -\left(b_2\cdot\frac{\partial V}{\partial x_2}+\frac{\partial V}{\partial x_3}\right)\cdot\sum_{j=1}^{3}\Delta k_ix_i=0. \quad (9)$$

Substituting (6) into (5), we obtain:

Implementing minimization procedure, we obtain expressions for the optimal values of control variables variations:

$$\Delta k_{iopt}=\left(\frac{b}{2c_i}\cdot\frac{\partial V}{\partial x_2}+\frac{1}{2c_i}\cdot\frac{\partial V}{\partial x_3}\right)\cdot x_i,\ i=\overline{1,3}. \quad (10)$$

After exclusion Δk_i and Δk_{iopt} we obtain a modification of the Hamilton – Jacobi - Bellman equation:

$$\sum_{i=1}^{3}\alpha_ix_i^2+\frac{\partial V}{\partial x_1}\cdot(-a_1x_1+a_2x_2)+ \\ +\frac{\partial V}{\partial x_2}\cdot(-a_3\cdot x_2+b_1\cdot x_3- \\ -b_2\cdot\sum_{i=1}^{3}k_ix_i)-\frac{\partial V}{\partial x_3}\sum_{i=1}^{3}k_ix_i=0. \quad (11)$$

Suppose that the solution of equation (11) is a quadratic form:

$$V(x_1,x_2,x_3)=\sum_{ij=1}^{3}k_{ij}x_ix_j\ . \quad (12)$$

Further ADSR procedure reduced to finding the coefficients of the quadratic form. Finally, the equation of the regulator in the general form will be:

$$\sigma_2'=-(\sum_{i=1}^{3}k_ix_i+\sum_{i=1}^{3}\frac{b_2}{c_i}x_i^2(\sum_{j=1}^{3}k_{j2}x_j)+ \\ +\sum_{i=1}^{3}\frac{1}{c_i}x_i^2(\sum_{j=1}^{3}k_{j3}x_j), \quad (13)$$

there k_{j2}, k_{j3} – coefficients of the quadratic form obtained by "stitching" solutions, $j=\overline{1,3}$; k_i – coefficients of the quadratic form, obtained with solving the problem "in small", $i=\overline{1,3}$.

Reseach of quality of proposed solution The research of dynamic characteristics of the armature current control loop with relay control, as well as synthesized nonlinear control was carried out by numerical modeling. In this case, the character of setpoint on the tension from an external controller was modeled as a step signal. Transients of armature current using relay (curve 1) and the synthesized nonlinear controller (curve 2) are shown in Figure 3.

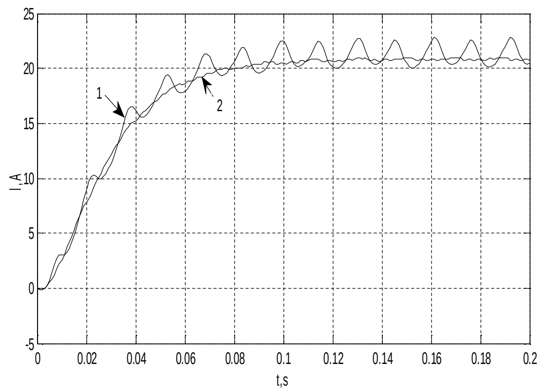

Fig. 3. Transients of armature current using different regulators

It can be seen that in the case of using relay regulators oscillation mode may occurs. When using a non-linear synthesized controller of armature current motor torque fluctuations, disappears.

Conclusions. Applying of the Bellman-Lyapunov method using invariant imbedding has significantly improved the quality of the transient process on the armature current, and therefore the torque of the paper tension of unwinding mechanism. So, we used the method of the "instantaneous" linearization and a replacement of relay with two positions with hysteresis by mathematical model with univocal nonlinearities. Further improving of transients quality is possible due to using of classical quadratic quality functional where solution of Gamilton-Yacobi-Bellman equation would be presented as a sequence of exponent forms. However, it will lead to a significant complication of the synthesis procedure and the need for including of high degree components of state variables into control law.

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