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GENERALIZED ARTINA HYPOTHESIS AND COMPUTER INFORMATION MODEL ITS SOLUTIONS

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Abstract. *In this paper, computer simulation methods are used to study the properties of primes as a dynamically developing system. It is shown that unsolved problems in the theory of numbers severely limit the further development of mathematical science in both theoretical and applied aspects. The foundations of experimental mathematics are formed as a tool for constructing information technologies in pure and applied number theory. On the basis of Artin's hypothesis, a generalized hypothesis is formed and shown as a means of non-linear dynamical processes to obtain a sufficiently accurate solution of it.*

Key word: *Gödel's theory; groups of residues; primitive root; distribution of prime numbers; recursion; Artin's hypothesis; estimation of Artin parameters.*

Introduction

In mathematics, until the thirties of the twentieth century, there was a belief that any problem in mathematics could be solved. However, in 1935 Gödel proved that if constructive mathematical theory includes arithmetic, then it always contains a true theorem, which is unprovable by means of the given mathematical theory [1]. Since this moment, active research has begun in the theory of recursive functions and effective computability [2]. In parallel, many unsolved mathematical problems have become the object of detailed research in terms of assessing the complexity of their solution. At the present time, a large number of mathematical problems are known about which there is no information on their solvability. In the field of modern number theory a large list of such problems with detailed analysis is given in monographs [3, 4] and a number of other papers. One of these problems is the Artin hypothesis [5] formed in 1927 and has not been solved so far.

An important problem in number theory is the description of the law of distribution of prime numbers. This problem was solved by Hadamard and Valle-Poussin, independently of each other, in 1896 [6]. They proved that the number of prime numbers $\pi(x)$ less than or equal to x is determined by the expression

$$\pi(x) = \int_2^x \frac{dt}{\ln t} + O\left(x \cdot e^{-\frac{c}{2}\sqrt{\ln x}} \right) \qquad (1)$$

where c is an absolute constant. This analytically proved form of representation of the law of distribution of prime numbers has already become universally recognized in the mathematical world. Yet two things should be noted. First, it was obtained on the basis of the analytic zeta-Riemann function, which, until it is proved, adequately describes the distribution of primes in a complex space. According to the Riemann hypothesis, all the zeros of the zeta function are on the line passing through the point equal to $1/2$. This millennium hypothesis has not yet been proved. And this fact is the basis for criticizing all the results obtained on the basis of the zeta-Riemann function.

The second circumstance is that simultaneously with this fact the dynamics of the change of $O\left(x \cdot e^{-\frac{c}{2}\sqrt{\ln x}} \right)$ [7] is investigated. In [8, 9], an estimate of the entropy of this estimate is obtained and it is proved that it is fractal in nature. These facts are the basis for the formation of proposals on the need to study other models for the distribution of prime numbers. Some results of such investigations are given in monographs [10, 11]. Another problem related to the distribution of prime numbers appeared in 1927, when the well-known mathematician Artin formed a hypothesis about the distribution of prime numbers for which the natural number $a > 1$ is given is their primitive root [12, 13].

According to the Artin conjecture [13], the set of such prime numbers has the distribution law $\pi(x, a)$ in the form of the expression

$$\pi(x, a) = c(a) \cdot \pi(x) \qquad (2)$$

where $\pi(x)$ is the distribution of primes, and $c(a)$ is a constant depending on a. So far, despite numerous studies, this hypothesis has not been resolved. At the same time, it is not known whether this is true for any values of a. If the hypothesis is correct, the question remains how to evaluate the constant $c(a)$ for each particular a and what properties of the number a affect its value. Answers to these questions are still lacking. In [14, 15] a detailed analysis of all research results in the field of the Artin hypothesis solution is given.

It should be noted that the proof of the Artin conjecture is important both from the theoretical point of view in number theory and from the application point of view, since its positive solution is important in cryptography, coding theory, and the theory of dynamical systems. In [16], a generalized Artin conjecture was formulated for any $a>1$, i.e. a may not be a primitive root. According to Artin's generalized theory, equality is valid

$$\pi(x,a,i)=c(a,i)\cdot\pi(x) \qquad (3)$$

where $a>1$, i is the index of the class of primes in the classification of prime numbers generated by the numbers a, $c(a,i)$ is a constant. According to the classification constructed in [16]

$$P_{a,i}=\left\{p\in P \mid \frac{(p-1)}{card_a(p)}=i\right\} \qquad (4)$$

where $card_a(p)$ is the length of the recursion $x_{n+1}\equiv ax_n(\operatorname{mod} p)$ for $x_0=1$.

It is not difficult to show that for any $a>1$ the equality

$$\sum_{i=1}^{\infty}c(a,i)=1 \qquad (5)$$

This means that prime numbers are uniformly distributed in classes $P_{a.i}$ for any a. By uniformity it is meant that within each class of primes $P_{a.i}$ the logarithmic law of distribution of primes is preserved. The constant $c(a,i)$ defines the measure of the uniform decimation of primes based on the value of a. If $i-1$ then a is the primitive root of all primes $P_{a.1}$.

The determination of $c(a,i)$ for any a,i by analytical methods is unlikely in the short term. However, the formation and development of experimental mathematics [17, 18] opens another way to solve this problem by computer modeling methods for nonlinear dynamic processes of prime numbers formation.

Simulation of dynamic processes of distribution of simple numbers in the system of classification by the module of prime numbers.

The process of modeling the dynamics of the formation of prime numbers was built on the following assumptions. Suppose that we are given an ordered set of prime numbers $P=\{p_1,p_2,\ldots,p_k,\ldots\}$ whose elements are ordered in ascending order. All this set was broken into a subset of 500,000 prime numbers. The number 500000 is due to the limitations of MS Excel, as a tool for statistical analysis, a number of characteristics of the process of forming prime numbers. Only one limitation is important. Always choose 500000 consecutive prime numbers from the set P. In the modern version of Excel, this number can be increased to one million. If you use a powerful computer, you can choose any larger number instead of a million.

The realized variant of the study of dynamic processes of the formation of prime numbers includes the following indicators: the number of the prime number p in the ordered set P, the value of the prime number p, the length of the recursion of the numbers $card_a(p)$ for the same value a for all primes P, the value $ind_a(p)$ of the index of the class, t .e. $ind_a(p)=(p-1)/card_a(p)$, the residue values modulo any natural module $n>1$, for all classes and any other analyzed properties of primes or factors of the decomposition of the number $p-1$ into prime factors. To each simple factor p_i in the decomposition $p-1=\prod_{i=1}^{n}p_i^{\alpha_i}$, one indicator of the dynamic process of the formation of prime numbers is put in correspondence, individual values that can be analyzed for any other indicators are the values for them of the residue modulo the natural number $n>1$. The only exception is $ind_a(p)$. The number of controlled indicators analyzed in the Excel environment can be expanded.

The basis for the emergence and development of experimental mathematics is the following iterative scheme for solving mathematical problems figure 1.

According to the idea of experimental mathematics, at the first iteration we start from hypothetically known data. But it is also the basis for obtaining experimental information on the basis by which analytical methods of number theory give an extended representation of the hypothesis in the form of H_i. It is possible that the hypothesis can be corrected or even rejected as not true. Refined from the point of view of information technology in mathematics, the hypothesis H_i is used to develop from the point of view of deepening experimental mathematics the model of in-depth studies at the I_1 level.

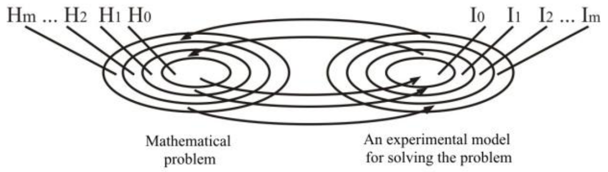

Figure 1. Scheme of the dynamic process of information technology formation to solve the problem

The iteration process is continued until an analytically valid solution of the generated hypothesis is obtained. Since the generalized Artin conjecture is considered in this paper, we present the results of estimating the constant $c(a,i)$ for the case $a=4$ and $i=2$. The number $a=4$ is a perfect square, and therefore it cannot be a primitive root. From the point of view of the generalized Artin conjecture, this is as interesting and important as in the case when $a=4$ is a primitive root.

Based on the data given in [16], estimates of $c(4,i)$ for $i=2,4,\ldots,20,..$ were obtained. It is shown that their values are stable for class $P_{4.2}$. class with $ind_4(p)=2$ to the fourth decimal place.

According to the hypothesis of Artin [12], the set of all prime numbers $\pi(x,a)$ for which the natural number a is their primitive root satisfies the equality:

$$\pi(x,a)=c(a)\cdot\pi(x)$$

where $c(a)$ is the Artin constant depending on a and their determining share on the set $\pi(x)$ of primes greater or equal. If x tends to infinity then in the limit at $x\to\infty$. The term share of primes can be replaced by the probability of choosing a prime number p from the set of all primes can be used when the set of natural numbers N is given a discrete uniform distribution law such that the probability distribution function $f(x)$ changes by one and the same amount only by simple numbers $p\in P$. In this case, infinite sets $P_{f(p)}$ consisting of all prime numbers p are considered such that the function $f(p)$ determines that p belongs to the set $P_{f(p)}$. In this case, the set $P_{f(p)}$ has a probability measure different from zero if this set is infinite. For finite sets of primes, we assume that their probability measure is zero due to the assumption that the probability distribution obeys a uniform law.

Due to the fact that a is the primitive root of all primes P, then the index $ind_a(p)=(p-1)/card_a(p)$ is equal to one according to Fermat's small theorem for all other primes $p\in P$ $ind_a(p)$ will be greater than one. If a is a complete square, then it is obvious that for such a the set of primes $p\in P$ with the same indices $ind_a(p)$ will have an index greater than 1. Therefore, the whole set of primes is divided into classes with the same index. We will denote $P_a.f(i)$ the set of all simple $p\in P$ such that:

$$P_a.f(i)=\{p\in P\,|\,ind_a(p)=(p-1)/card_a(p)=f(i)\},$$

where i indicates the ordinal number of this subset when ordering such subsets of primes with the same index by the value of the index. Given such a partition of P into non-intersecting subsets of $P_a.f(i)$, the generalized Artin's hypothesis is true, which is equal to

$$\pi(x,a,f(i))=c(a,f(i))\pi(x)$$

$$\sum_{i=1}^{\infty}c(a,f(i))=1 \text{ for any } a>1.$$

Note that a may not be a primitive root for one prime number p. This is true when a is a perfect square and $a>1$. In all such cases, $c(a,f(1))=0$ and $c(a,f(i))$ can be equal to zero for other subsets of primes in the base, and with large values of $ind_a(p)$.

From this formulation of the generalized Artin hypothesis, it follows the assumption that the logarithmic law of the distribution of primes is valid for any $a>1$ for all sets of primes.

$$P_{a.f(1)},P_{a.f(2)},\ldots,P_{a.f(x)},\ldots$$

and at the same time there is a probability measure P_a such that $P_a(P_a.f(i))=c(a.f(i))$. This point of view is close in meaning to the Erdos-Kac theorem [20] according to

$$\lim_{n\to\infty}\frac{1}{N}\left|\left\{1\le n\le N\,|\,a\le\frac{w(n)-\ln\ln N}{\sqrt{\ln\ln N}}\le b\right\}\right|=\frac{1}{\sqrt{2\pi}}\int_a^b e^{-\frac{x^2}{2}}dx$$

where $w(n)$ is the number of prime divisors of n without taking into account their degree in the decomposition of n into prime factors.

In this paper, we consider the Artin hypothesis only over the set of prime numbers P. A generalization to the case of finite fields and elliptic curves is not considered. It should be noted that the above generalization can be moved to the case of finite fields, elliptic curves.

To solve the generalized, in the above sense, Artin's hypothesis, we chose a method of computer modeling, which is given in our papers [21].

The entire set of P primes is divided into subsets of 500000 consecutive primes. The object of the analysis was the first ten million prime numbers. The developed method and software product allows both to expand the volume of analyzed consecutive prime numbers, and to shift blocks of 500,000 by any value. The number of consecutive primes in the block can be increased to any final value, the number 500000 is chosen for two reasons. First, such a quantity is sufficient to obtain estimates of the Artin constants for primitive roots with an accuracy of three decimal places. Their successive integration is possible to the level of blocks of a million or more prime numbers.

In the process of modeling the formation of classes $P_a.f(i)$ $i=1,2,...,k,..$, classes of primes for all primes from the first ten million prime numbers were distinguished and the exact number of primes in each of the classes was calculated. Since the logarithmic law of distribution of primes is valid on every sufficiently large subset of consecutive primes and at the same time on Artin’s hypothesis and its generalized form, it follows that its extension to any set of consecutive primes of any size not lower than the above, but located outside of the investigated interval will keep the obtained estimates.

It has been established that in order to preserve the accuracy of estimates or increase the value of accuracy when moving to intervals of consecutive primes, it is necessary to increase the length of the interval. This is due to the fact that with an increase in the prime numbers the distance between them increases. The dependence of the accuracy of the estimate $c(a, f(i))$ on the values of the maximum distances between prime numbers. The study of this dependence is a separate complex mathematical problem.

To obtain estimates of the $c(a, f(i))$ constants in Artin’s generalized hypothesis, a computational analysis of the class structures was performed:

$$P_a.f(1), P_a.f(2), P_a.f(3), ..., P_a.f(n), ...$$

for $a \in \{2,3,4,...,15,16\}$ values on the set of the first ten million prime numbers. In this case, $f(i)=i$, although in the future we will consider other classes of primes and the function $f(i)$ will determine the index of the i-th subset of primes in the whole set of primes whose elements satisfy the given conditions. In the case of the generalized Artin's hypothesis, the following equality takes place:

$$\begin{cases} P_a.f(i) = P_{a.i} = \{p \in P \mid ind_a(p) = ((p-1)/cord_a(p)) = i\}; \\ c(a.f(i)) = c(a.i) = |P_a.f(i)|/10^7 = |P_{a,i}|/10^6; \\ \sum_{i=1}^{10^6} c(a.f(i)) = \sum_{i=1}^{10^6} c(a.i); \end{cases} \qquad (6)$$

The chosen method for solving the generalized Artin hypothesis consists of two stages:

– computational experiment for a given set of consecutive prime numbers of power M. In this case, the first ten million prime numbers are chosen, and therefore $M = 10^6$.

– based on analytical number theory, to prove the correctness of estimates obtained on the basis of a computational experiment in accordance with relations (6).

Table 1 shows the results of a computational experiment. For the numbers 2, 3, 6, 7, 10, 11, 12, 15, all $c(a_j, i)$ constants for $j \in \{2,3,6,7,10,11,13,15\}$ coincide almost to the accuracy of two and even three decimal places. Ambrose C. D. [19] formed the assumption that this is true for sets of prime numbers for which 2, 3 are primitive roots, but the proposition is not proved. On the basis of the obtained data, it can be argued that this is true for a wider set of primitive roots. Interestingly, the set of primes with primitive roots 5 and 8 is not fulfilled. Especially significant differences exist for the number $a=8$.

In the case of $a=8$, the generalized constants $c(8,i)$ satisfy the condition $\sum_{i=1}^{\infty} c(8,i) = 1$, but their values have a different dynamics of change for many classes.

$$\{P_{8.1}, P_{8.2}, ..., P_{8.k}, ...\}$$

In this paper, the cause of a radical change in the values of Artin's constants is not investigated. For the number $a=5$, the value of the Artin constant for the set of primes for which $a=5$ is a primitive root, the same has a different meaning.

Table 1

The distribution of prime numbers in 1 to 16 classes in the generalized Artin’s conjecture

c(a,i)	$P_{a,1}$	$P_{a,2}$	$P_{a,3}$	$P_{a,4}$	$P_{a,5}$	$P_{a,6}$	$P_{a,7}$	$P_{a,8}$
$P_{2,i}$	0,374	0,280	0,066	0,047	0,019	0,050	0,009	0,035
$P_{3,i}$	0,374	0,299	0,067	0,056	0,019	0,033	0,009	0,014
$P_{4,i}$	0,000	0,561	0,000	0,094	0,000	0,100	0,000	0,070
$P_{5,i}$	0,394	0,266	0,070	0,066	0,000	0,047	0,009	0,017
$P_{6,i}$	0,374	0,280	0,066	0,075	0,019	0,050	0,009	0,014
$P_{7,i}$	0,374	0,283	0,066	0,068	0,019	0,050	0,009	0,017
$P_{8,i}$	0,224	0,168	0,199	0,028	0,011	0,150	0,005	0,021
$P_{9,i}$	0,000	0,598	0,000	0,112	0,000	0,067	0,000	0,028
$P_{10,i}$	0,374	0,280	0,067	0,071	0,019	0,050	0,009	0,017
$P_{11,i}$	0,374	0,281	0,066	0,069	0,019	0,050	0,009	0,017
$P_{12,i}$	0,374	0,299	0,066	0,056	0,019	0,033	0,009	0,014
$P_{13,i}$	0,376	0,279	0,067	0,070	0,019	0,050	0,009	0,017
$P_{14,i}$	0,374	0,281	0,066	0,071	0,019	0,050	0,009	0,017
$P_{15,i}$	0,374	0,280	0,066	0,071	0,019	0,051	0,009	0,018
$P_{16,i}$	0,000	0,374	0,000	0,187	0,000	0,066	0,000	0,140

But this is not the main non-standard behavior of $a = 5$. It is important that the classes of primes $\{P_{5,5}, P_{5,15}, P_{5,25}, ..., P_{5,10k+5}, ...\}$ are empty sets. There are other deviations from the standard of conduct for $a \in \{2,3,6,7,10,11,14,15\}$. The analysis of such deviations requires specific targeted research.

Conclusions

A generalized Artin's hypothesis is formed in this paper Based on the methods of experimental mathematics and the developed computer simulation methods for a given set of natural numbers $a = \{2,3,4,5,...,14,15,16\}$ it was established that Artin’s generalized hypothesis is true. The estimates of the Artin constants are obtained. The concept of analytical proof of the generalized Artin's hypothesis is formed.

The results of experimental mathematics in table 1 of the first iteration confirm that Artin's conjecture is correct. Estimates of the constants are obtained to within a third decimal place. The data of the table confirm the generalized Artin hypothesis for $a = \{2,3,4,5,...,14,15,16\}$ and the assumption that $\sum_{i=1}^{\infty} c(a,2=i) = 1$. The obtained results are the basis for constructing an analytic proof of the Artin conjecture and its generalization.

The paper presents the foundations of a new approach to research in the theory of numbers and dynamical systems starting with prime numbers. It is shown that the development of methods of computer modeling creates the basis for creating iterative procedures for solving complex mathematical problems in combination:

<computer model> $\Leftrightarrow$ <analytical theory>.

It is established that in mathematical theories in the study of dynamic processes it is necessary to take into account the properties of objects of sets over which the mathematical theory is constructed.

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УЗАГАЛЬНЕНА ГІПОТЕЗА АРТІНА ТА КОМП'ЮТЕРНА ІНФОРМАЦІЙНА МОДЕЛЬ ЇЇ ВИРІШЕННЯ

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Анотація. *В роботі методами комп'ютерного моделювання досліджені властивості простих чисел, як динамічної системи, що розвивається. У теорії чисел сформувалися такі напрямки досліджень: аналітичне, алгебраїчне, обчислювальне, комбінаторне, геометричне, ймовірнісний. Незважаючи на їх різноманіття і величезна кількість публікацій кількість не вирішених проблем вельми вражає і збільшується безперервно. У дослідженнях з теорії чисел значні зусилля пов'язані з простими і натуральними числами. Показано, що невирішені проблеми теорії чисел істотно обмежують подальші розвиток математичної науки як теоретичному, так і прикладному аспектах. Сформовані основи експериментальної математики, як інструменту побудови інформаційних технологій в чистої та прикладної теорії чисел. На основі гіпотези Артіна сформована узагальнена гіпотеза і показано як засобами нелінійних динамічних процесів отримано достатньо точне її рішення. В математиці до тридцятих років XX століття існувало переконання, що будь-яка проблема в математиці може бути вирішена. Однак в 1935 році Гедель довів, що якщо конструктивна математична теорія включає арифметику, то в ній завжди знаходиться справжня теорема, яка недовідна засобами даної математичної теорії. Починаючи з цього моменту, почалися активні дослідження в теорії рекурсивних функцій і ефективної обчислюваності. Паралельно багато невирішених математичні проблеми стали об'єктом детального дослідження з точки зору оцінки складності їх вирішення. У теперішній час відомо велика кількість математичних проблем, щодо яких, відсутня яка-небудь інформація щодо їх розв'язання. В області сучасної теорії чисел великий перелік таких проблем з детальним аналізом наведено в монографіях і ряді інших статей. Однією з таких проблем є гіпотеза Артіна сформована в 1927 році і не вирішена досі. Важливою проблемою* *теорії чисел є опис закону розподілу простих чисел. Дане завдання було вирішене Адамаром і Валле-Пусеном, незалежно один від одного, в 1896 році.*

Ключові слова: *Теорія Геделя, групи відрахувань, первісний корінь, розподіл простих чисел, рекурсія, гіпотеза Артіна, оцінювання параметрів Артіна.*

ОБОБЩЕННАЯ ГИПОТЕЗА АРТИНА И КОМПЬЮТЕРНАЯ ИНФОРМАЦИОННАЯ МОДЕЛЬ ЕЕ РЕШЕНИЯ

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Аннотация. *В работе методами компьютерного моделирования исследованы свойства простых чисел, как динамической развивающейся системы. Сформированы основы экспериментальной математики, как инструмента построения информационных технологий в чистой и прикладной теории чисел. На основе гипотезы Артина сформирована обобщенная гипотеза и показано достаточно точное ее решение.*

Ключевые слова: *Теория Геделя, группы вычетов, первообразный корень, распределение простых чисел, рекурсия, гипотеза Артина, оценивание параметров Артина.*

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